Algebra I, Quarter 2, Unit 2.1

Line of Best Fit

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent data using two variables on a scatter plot.
- Interpret the slope as a rate of change.
- Interpret the intercept of a linear model related to data.
- Using technology, compute and interpret the correlation coefficient of a linear fit (line of best fit).
- Using technology, predict past and future outcomes in the context of the data.
- Distinguish between correlation and causation.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use scales and units to make graphs that appropriately represent data.
- Make conjectures about the form and meaning of solutions.
- Search for regularity or trends.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.

Construct viable arguments and critique the reasoning of others.

- Reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Model with mathematics.

- Solve real-world mathematics problems using linear expressions.
- Identify important quantities in a practical situation and map their relationships using such tools as graphs and formulas.
- Analyze relationships mathematically to draw conclusions.

Essential questions

- How can we use data to make predictions about the future?
- How do you explore the relationship between variables?
- How do you determine the line of best fit using linear regression for a set of data points using various methods, including eyeballing and technology?
- What is the slope-intercept form and how can it be used to draw the line it represents?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Categorical and Quantitative Data★	**S-ID**

Summarize, represent, and interpret data on two categorical and quantitative variables. *[Linear focus, discuss general principle]*

S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S-ID.9 Distinguish between correlation and causation.★

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In kindergarten and first grade, students gathered and compared data. In second grade, they drew pictographs and bar graphs with single-unit scales of up to four categories. Third-grade students scaled pictographs and bar graphs and transferred data to line plots. In the fourth and fifth grades, students used line plot with fractions (1/2, 1/4, and 1/8). Sixth-grade students displayed numerical data in plots on a number line, including dot plots, histograms, and box plots. In seventh grade, students began to graph ordered pairs. Eighth-grade students interpreted the equation $y = mx + b$ as a linear function.

Current Learning

Algebra I students represent data in a scatterplot and interpret the slope and intercept of a linear model in the context of the data. Students also use technology to compute and interpret the correlation coefficient of a linear fit. Students distinguish between correlation and causation.

Future Learning

In Algebra II, students will summarize, represent, and interpret data on a single count or measurement variable. Students will apply this content to real-world connections and careers, including risk assessment analysts, accountants, business owners, statisticians, financial managers, engineers, economists, and in the fields of advertising and medicine.

Additional Findings

This unit builds upon students' prior experiences with data, providing them with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit. (CCSS Appendix A)

Algebra I, Quarter 2, Unit 2.2

Solving Systems of Equations

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Solve systems of linear equations exactly and approximately.
- Explain why the *x*-coordinate is the source in determining the *y*-coordinate of the equation (substitution).
- Graph the solution of linear inequalities as a half plane.
- Graph the solution of systems of linear inequalities as the intersection of the corresponding half planes.
- Find viable and nonviable solutions to systems of equations and inequalities.
- Solve systems of equations using substitution.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- Make conjectures and build a logical progression of statements to explore the truth of the conjectures.

Model with mathematics.

- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, graphs, and formulas.
- Solve real-world math problems using linear systems.
- Analyze relationships mathematically to draw conclusions.

Use appropriate tools strategically.

- Use calculators or dynamic geometry software.
- Analyze graphs of functions and solutions generated using a graphing calculator.
- Use technological tools to explore and deepen understanding of concepts.

Essential questions

- How can linear functions be used to model problem situations?
- What does a solution of a system of linear equations mean?
- Why it is sometimes easier to solve equations using substitution rather than graphing?
- What are the differences in graphing linear inequalities and linear equations?
- Why are the *x*-coordinates of a point essential for identifying the *y* values of a function?

Written Curriculum

Common Core State Standards for Mathematical Content

Reasoning with Equations and Inequalities	**A-REI**

Solve systems of equations [*Linear-linear ~~and linear-quadratic~~*]

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically [*Linear ~~and exponential~~; learn as general principle*]

A-REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value, exponential, and logarithmic~~ functions.★

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Creating Equations★	**A-CED**

Create equations that describe numbers or relationships [*Linear, ~~quadratic, and exponential~~ (integer inputs only); for A.CED.3 linear only*]

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In first and second grades, students started solving addition and subtraction equations with unknowns in various positions. They solved multiplication and division equations of this sort in grades 3 and 4. In fourth grade, students used all four operations to solve problems with unknowns. Sixth-grade students solved one-variable equations, and in seventh grade, they began solving real-world equations. Eighth-graders solved pairs of linear equations.

Current Learning

Algebra I students solve systems of linear equations exactly (substitution) and approximately (graphically). Students also represent and solve equations and inequalities graphically. Students use technology to graph systems of equations and inequalities on graphing calculators. They represent constraints by equations and inequalities and by systems of equations and/or inequalities, and they interpret solutions as viable or nonviable options in a modeling context.

Future Learning

In Algebra II, students will create equations that describe numbers or relationships, including equations using all available types of expressions and including simple root functions. They will represent and solve equalities and inequalities graphically, combining polynomial, rational, radical, absolute value, and exponential functions. This content has applications in many careers, include statistics, engineering, economics, and nursing.

Additional Findings

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities and using them to solve problems. They master the solution of linear equations and apply related solution techniques. (CCSS Appendix A)

Algebra I, Quarter 2, Unit 2.3

Introducing Quadratics

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions.
- Identify ways to rewrite expressions.
- Factor a quadratic expression to reveal zeros of the function it defines.
- Complete the square in a quadratic expression to reveal maximum or minimum values.
- Use exponential functions to calculate interest earned.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

Construct viable arguments and critique the reasoning of others.

- Justify conclusions, communicate to others, and respond to the arguments of others.

Model with mathematics.

- Apply the mathematics known to solve problems arising in everyday life, society, and the workplace.
- Interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Essential questions

- What can you learn from parts of a quadratic expression, such as terms, factors, and coefficients?
- What is meant by the simplest form of a quadratic expression?
- Why does a quadratic equation have two solutions?
- What are the steps to be taken in factoring a quadratic expression?
- What steps are to be taken in completing the square of a quadratic?
- Why is it important to be able to calculate interest on loans and /or savings?

Written Curriculum

Common Core State Standards for Mathematical Content

Seeing Structure in Expressions	A-SSE

Interpret the structure of expressions [~~*Linear, exponential,*~~ *quadratic*]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems [*Quadratic* ~~*and exponential*~~]

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

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3 Construct viable arguments and critique the reasoning of others.

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Clarifying the Standards

Prior Learning

In sixth grade, students began to find the greatest common factor of two whole numbers. Seventh-graders applied properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Students in eighth grade solved linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Current Learning

Students review and expand on their skills at interpreting parts (such as terms, factors, and coefficients) of an expression. They also interpret complicated expressions by viewing one or more of their parts as a single entity. The structure of an expression is used to identify ways to rewrite it. Students factor quadratic expressions to reveal the zeros of the functions they define. Students complete the square in a quadratic expression to reveal the maximum or minimum of the function it defines.

Future Learning

In Algebra 2, students will interpret the structures of polynomial and rational expressions and write expressions in equivalent forms to solve problems. Connections to the real world and careers include software designers, engineers, economists, insurance underwriters, purchasing managers, and buyers.

Additional Findings

It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions—for example, development of skill in factoring and completing what the different forms of a quadratic expression reveal. In this unit, students build on their knowledge of extending the laws of exponents to rational exponents. Students apply this new understanding of numbers and strengthen their ability to see structure in quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. (CCSS Appendix A)